



ELSEVIER

Economics Letters 66 (2000) 85–91

**economics
letters**

www.elsevier.com/locate/econbase

The preferred hedge instrument

Harald L. Battermann^a, Michael Braulke^b, Udo Broll^{c,*}, Jörg Schimmelpfennig^d

^aDepartment of Economics, Technische Universität Chemnitz, D-09107 Chemnitz, Germany

^bDepartment of Economics, Universität Osnabrück, D-49069 Osnabrück, Germany

^cDepartment of Economics, Universität Bonn, D-53113 Bonn, Germany

^dDepartment of Economics, Ruhr-Universität Bochum, D-44780 Bochum, Germany

Received 30 December 1998; accepted 27 July 1999

Abstract

The optimal choice of hedging instruments in futures and option markets is analyzed for a risk averse exporting firm that maximizes expected utility. Assuming unbiased futures and options prices, optimal output and hedging decisions are derived. It is shown that futures will unequivocally be preferred to options. This preference for futures continues even if their price is adversely biased, provided the bias is not too strong. © 2000 Elsevier Science S.A. All rights reserved.

Keywords: Exchange rate risk; Futures; Options

JEL classification: F30; G15

1. Exports, futures and options

The aim of this note is to study the optimal choice of hedging instruments of an exporting firm exposed to exchange rate risk when either currency futures or options are available. In the presence of unbiased futures and options prices, it is shown that the hedge effectiveness of futures is greater than that of options. With currency futures a maximal hedging effectiveness occurs at a full hedge strategy.

The plan of this note is as follows. First we present the model and compare the optimal output and hedging decisions under the two alternatives. We then answer the question which of the two hedging instruments the firm prefers and conclude the paper by brief remarks.

Consider a firm producing a final good X at increasing marginal cost, $C'(X) > 0$, $C''(X) > 0$, for an export market that faces a random foreign exchange rate S . The world price P is given. The spot

*Corresponding author. Present address: Department of Economics, Saarland University, D-66041 Saarbrücken, Germany. Tel.: +49-681-302-4353; fax: +49-681-302-4390.

E-mail address: broll@iwb.uni-sb.de (U. Broll)

exchange rate S is random with a known density function. It is assumed that the firm is risk averse and maximizes a von Neumann–Morgenstern utility function $U(Y)$ with $U' > 0$ and $U'' < 0$, where Y denotes uncertain terminal net income.

It is assumed that the firm may choose among entering either a currency futures or an options market. However, only futures and put options will be compared here, because the case of call options would be entirely symmetrical to that of puts, whereas the combined case of allowing simultaneous access to puts and calls would, in turn, give rise to a synthetic futures contract (Broll and Wahl, 1992). Thus, we will ignore calls in the following analysis.

1.1. Currency futures

Consider first the case where the firm may use a competitive currency futures markets. Let F denote the given forward price at date 0 for the delivery of one unit of foreign exchange at date 1.¹ Just as the output level X , the volume of the futures contract Z_f has to be determined by the firm ex ante. It chooses X_f and Z_f so as to maximize the expected utility of final income Y_f , i.e., $EU(Y_f)$, where E is the expectation operator with respect to the distribution function of S and

$$Y_f = SPX_f - C(X_f) + Z_f(F - S).$$

Since the firm is, by assumption, risk-averse, the maximand is strictly concave in X and Z . Hence, the first order conditions

$$EU'(Y_f^*)(SP - C'(X_f^*)) = 0, \quad (1)$$

$$EU'(Y_f^*)(F - S) = 0, \quad (2)$$

are both necessary and sufficient for a maximum. In what follows we will assume that the futures market is unbiased, i.e.,

$$F = E(S). \quad (3)$$

In this case, (2) reduces to²

$$\text{cov}(U'(Y_f^*), F - S) = 0, \quad (4)$$

implying that Y_f^* has to be constant since $F - S$ is a strictly decreasing function of S . Hence, $Z_f^* = PX_f^*$, for then

$$Y_f^* = FPX_f^* - C(X_f^*) \quad (5)$$

is indeed constant. The optimal hedging volume is thus a full hedge.

It remains to establish the output decision X_f^* . Given the constancy of Y_f^* , (1) reduces to

¹For simplicity, we ignore interest aspects, i.e., assume a zero rate of interest.

²Note that by definition of a covariance $EAB = EAEB + \text{cov}(A, B)$.

$E(SP - C'(X_f^*)) = 0$, which in view of (3) implies that the firm chooses the output level at which $C'(X_f^*) = FP$, i.e., where marginal cost equal the expected (domestic currency) price.³

1.2. Currency options

Consider, alternatively, the case where the firm may use a currency options market. Let Z_p denote its volume of European put options under strike price K with payoff $\max\{K - S, 0\}$ and let R denote the put option price. With a currency put option, the net income of the firm is

$$Y_p = SPX_p - C(X_p) + Z_p [\max\{K - S, 0\} - R], \quad (6)$$

and the first order conditions are

$$EU'(Y_p^*)(SP - C'(X_p)) = 0, \quad (7)$$

$$EU'(Y_p^*)(\max\{K - S, 0\} - R) = 0. \quad (8)$$

Now assume that the options market is *fair*, i.e.,

$$E \max\{K - S, 0\} = R. \quad (9)$$

First order condition (8) then reduces to

$$\text{cov}(U'(Y_p^*), \max\{K - S, 0\} - R) = 0, \quad (10)$$

which implies $Z_p^* > PX_p^*$, i.e., the firm covers more than its foreign currency revenue with options.

We will prove this assertion by contradiction. Note first that

$$Y_p^* = \begin{aligned} & S(PX_p^* - Z_p^*) + (K - R)Z_p^* - C(X_p^*) \\ & \text{for } S < K, \end{aligned} \quad (11)$$

$$Y_p^* = \begin{aligned} & SPX_p^* - RZ_p^* - C(X_p^*) \\ & \text{for } S \geq K, \end{aligned} \quad (12)$$

and suppose, for the moment, that $Z_p^* \leq PX_p^*$. Income Y_p^* is then a rising function of S with, at most, a constant stretch over the range $(0, K)$. In view of the concavity of the utility function, $U'(Y_p^*)$ is, therefore, a falling function of S , and since $\max\{K - S, 0\} - R$ is so, too, their covariance must be strictly positive, contradicting (10). Hence, $Z_p^* > PX_p^*$ must hold. Thus, Y_p^* will fall until $S = K$ and rise thereafter, which means that $U'(Y_p^*)$ is first a rising and, from K on, a falling function of S .

³More generally, the output decision displays, of course, the well-known separation property, i.e., it is independent of the shape of the utility function and thus the hedging decision.

Given this information on the shape of $U'(Y_p^*)$, we may now determine the optimal choice X_p^* . Rewriting the first-order condition (7) as

$$EU'(Y_p^*)E(SP - C'(X_p^*)) + \text{cov}(U'(Y_p^*), SP - C'(X_p^*)) = 0, \quad (13)$$

it is evident that $E(SP - C'(X_p^*))$ and $\text{cov}(U'(Y_p^*), SP - C'(X_p^*))$ have opposite signs. Now, it can be shown that this covariance is negative.⁴ $E(SP - C'(X_p^*))$ is consequently positive which in view of (3) implies $C'(X_p^*) < FP$ and, by the convexity of the cost function, $X_p^* < X_f^*$. Thus, given the firm's inability to completely eliminate income uncertainty with options it will also use its output decision to reduce exposure to exchange rate risk by choosing a smaller volume.

2. The preferred choice

We are now in a position to answer our question whether the firm would choose fair priced options or futures for hedging purposes. Given $X_p^* < X_f^*$, it is easy to see that $EY_p^* < EY_f^* = Y_f^*$. Note, first, that by (6), (9) and (3) expected income in the options case amounts to $EY_p^* = FPX_p^* - C(X_p^*)$. Observe further that $\partial EY_p^* / \partial X = FP - C'(X_p^*) > 0$ because C' is increasing in X by assumption and $FP - C'(X)$ vanishes only at the larger output volume X_f^* . Thus, $EY_p^* = FPX_p^* - C(X_p^*) < FPX_f^* - C(X_f^*) = Y_f^*$. In terms of utility we have, therefore,

$$EU(Y_p^*) < EU(E(Y_p^*)) < EU(Y_f^*), \quad (14)$$

where the first inequality follows from Jensen's inequality, given the concavity of U and the stochastic nature of Y_p^* .

Our exporter, therefore, has two good reasons not to enter the options market and rely exclusively on futures, if both are fair priced. First, using the options market, expected income EY_p^* is smaller than the income realized when relying on futures. And second, the larger income from using futures is certain whereas the income using options remains stochastic.

3. Concluding remarks

In a standard one-period expected utility framework and with an unbiased futures or options market, it turned out that a risk averse firm would prefer access to a futures market. All else equal, the risk averse decision maker prefers the hedging instrument which best reduces the variance of future

⁴The proof is somewhat involved which is why we give it in Appendix A. There it is shown that $\text{cov}(U', K - S)$ is positive, which proves our point since this covariance and $\text{cov}(U', SP - C')$ must have opposite signs.

income. Choosing a full-hedge in the futures market makes terminal income deterministic, whereas the use of options would always leave an element of uncertainty. And here, on top of that, the certain income originating from the use of futures exceeds expected income with options. The strong chain of inequalities in (14) and a continuity argument suggests that the firm would even prefer adversely biased futures to fair priced options provided, of course, this bias is not too strong.

In our model, the random exchange rate entered the income equations linearly. When relaxing this assumption of a linear cash flow, the decision problem would fundamentally change and possibly create a genuine role for options as a hedging instrument (Froot et al., 1993; Lence et al., 1994; Moschini and Lapan, 1995).

Appendix A

To simplify notation, define $X(S) = U'(S)$, $Y(S) = \max(K - S, 0)$ and $Z(S) = K - S$ and the domains $A = [0, K]$ and $B = [K, \infty)$. Obviously, A and B span the entire domain of S without overlapping. Hence, we may express the unconditional expectation as a weighted sum of conditional expectations, e.g.,

$$E(X) = E(X|A)P(A) + E(X|B)P(B), \quad (\text{A.1})$$

where $P(A) = \int_A f(S) dS$ with f as the density of S and $P(B) = 1 - P(A)$. We will write $g(S) = f(S)/P(A)$ and $h(S) = f(S)/P(B)$ for the conditional densities on A and B , respectively.

Consider first $\text{cov}(X, Y)$ which, because of fair pricing of options, must vanish. Thus, we may write

$$\begin{aligned} 0 &= \text{cov}(X, Y) = E(XY) - E(X)E(Y) \\ &= E(XY|A)P(A) + E(XY|B)P(B) \\ &\quad - E(X)E(Y|A)P(A) - E(X)E(Y|B)P(B) \\ &= E(XY|A)P(A) - E(X)E(Y|A)P(A) \\ &= E[(X - E(X))Y|A]P(A), \end{aligned} \quad (\text{A.2})$$

because Y vanishes over B . Next we will demonstrate, by contradiction, that

$$E(X|A) - E(X) > 0. \quad (\text{A.3})$$

Suppose to the contrary that $E(X|A) - E(X) \leq 0$. Define $M \in A$ as the point in A where $X = E(X)$ ⁵ and denote $Y_M = Y(M)$. Clearly, $Y_M > 0$ because Y is strictly positive over the entire range of A . Note that,

⁵For K small enough, X may exceed $E(X)$ throughout all of A so that no M exists where $X = E(X)$. In this case set $M = 0$.

by construction, $X < E(X)$ and $Y > Y_M$ for all $S < M$, whereas $X > E(X)$ and $Y < Y_M$ for all $S > M$ because X is a strictly rising and Y a strictly decreasing function of S in A . Hence, we would have

$$\begin{aligned}
0 &\geq [E(X|A) - E(X)]Y_M \\
&= \int_A [(X - E(X))Y_M]g(S) dS \\
&= \int_0^M [(X - E(X))Y_M]g(S) dS \\
&\quad + \int_M^K [(X - E(X))Y_M]g(S) dS \\
&> \int_0^M [(X - E(X))Y]g(S) dS \\
&\quad + \int_M^K [(X - E(X))Y]g(S) dS \\
&= E[(X - E(X))Y|A],
\end{aligned} \tag{A.4}$$

where the strict inequality comes from the fact that the negative left-hand differences $X - E(X)$ are multiplied by $Y > Y_M$ while the positive right-hand differences are multiplied by $Y < Y_M$. However, in view of (A.2) and given $P(A) > 0$, the last line of (A.4) must vanish. Thus, we have a contradiction, and (A.3) holds which, in turn, implies

$$E(X|B) - E(X) < 0. \tag{A.5}$$

Consider eventually $\text{cov}(X,Z)$. Using again (A.1) we may write

$$\begin{aligned}
\text{cov}(X,Z) &= E(XZ) - E(X)E(Z) \\
&= E(XZ|A)P(A) + E(XZ|B)P(B) \\
&\quad - E(X)E(Z|A)P(A) - E(X)E(Z|B)P(B) \\
&= E(XZ|B)P(B) - E(X)E(Z|B)P(B) \\
&= E[(X - E(X))Z|B]P(B),
\end{aligned} \tag{A.6}$$

where the terms conditional on A vanish in view of (A.2) since $Y = Z$ on A . Now, define $N \in B$ as the point in B where $X = E(X)$ and write $Z_N = Z(N)$. Clearly, $Z_N < 0$ since $Z(K) = 0$ and Z is falling over the entire range of B . Note that, by construction, $X > E(X)$ and $Z > Z_N$ for all $S < N$, whereas $X < E(X)$ and $Z < Z_N$ for all $S > N$ because X and Z are strictly decreasing functions of S on B . To sign (A.6), multiply (A.5) by the constant $Z_N < 0$ to get

$$\begin{aligned}
 0 &< [E(X|B) - E(X)]Z_N \\
 &= \int_{B} [(X - E(X))Z_N]h(S) dS \\
 &= \int_{K} [(X - E(X))Z_N]h(S) dS \\
 &\quad + \int_{N} [(X - E(X))Z_N]h(S) dS \tag{A.7} \\
 &< \int_{K} [(X - E(X))Z]h(S) dS \\
 &\quad + \int_{N} [(X - E(X))Z]h(S) dS \\
 &= E[(X - E(X))Z|B] = \text{cov}(X,Z)/P(B).
 \end{aligned}$$

Here, the inequality sign strengthens again because, when replacing the constant Z_N by Z , both the negative products, $(X - E(X))Z$, on the left and the positive products on the right increase. Thus

$$\text{cov}(X,Z) = \text{cov}(U'(S),K - S) > 0. \tag{A.8}$$

References

Broll, U., Wahl, J.E., 1992. Hedging with synthetics, foreign-exchange forwards, and the export decision. *Journal of Futures Markets* 12, 511–517.

Froot, K.A., Scharfstein, D.S., Stein, J.C., 1993. Risk management: coordinating corporate investment and financing policies. *Journal of Finance* 48, 1629–1658.

Lence, S.H., Sakong, Y., Hayes, D.J., 1994. Multiperiod production with forward and option markets. *American Agricultural Economic Journal* 76, 286–295.

Moschini, G., Lapan, H., 1995. The hedging role of options and futures under joint price, basis, and production risk. *International Economic Review* 36, 1025–1049.